

## Question 1

①(a) Integration (Variation of Parameters)

$$y' + (\tan x)y = \sec x$$

Solution

$$\frac{dy}{dx} + (\tan x)y = \sec x \Rightarrow 0$$

$$dy + (\tan x)y dx - \sec x dx = 0$$

Integrating both sides

$$\int dy + \int \tan x y dx - \int \sec x dx = 0$$

$$y + y \int \tan x dx - \int \sec x dx = 0$$

$$y + (-y) |\cos x| - |\sec x + \tan x| = 0$$

$$\Rightarrow y - y |\cos x| - |\sec x + \tan x| = 0$$

$$y(1 - |\cos x|) - |\sec x + \tan x| = 0$$

$$\Rightarrow y = \frac{|\sec x + \tan x|}{1 - |\cos x|}$$

Question 2B)

Given  $y'' - \frac{(t+1)}{t} y' + y/t = t$ , let  $M = y'$ ,  $M' = y''$

$$\text{So } M' - \frac{(t+1)}{t} M + y/t = t$$

$$M' t - (t+1)M + y = t^2$$

$$\Rightarrow M' t - (t+1)M = t^2 - y$$

The ~~inter~~ equation is in the form  $ax^2 + bx = c$ , which is not required, then, we divide both sides by  $t$ ,

$$M' - \frac{t+1}{t} M = \frac{t^2 - y}{t} \text{ which is in the correct format of } u' + p(t)u = g(t), \text{ where } g(t) = \frac{t^2 - y}{t}$$

The integrating factor is  $\mu(t) = e^{\int p(t) dt}$ ,

$$\Rightarrow \mu(t) = e^{\int -\frac{(t+1)}{t} dt}, \mu(t) = e^{-\int (1 + 1/t) dt} = e^{-t + \ln|t|}$$

$$\text{Solve for } M = \frac{\int \mu(t) g(t) dt}{\mu(t)} + C$$

$$= \frac{\int e^{-(t + \ln|t|)} \cdot \left(\frac{t^2 - y}{t}\right) dt}{e^{-(t + \ln|t|)}} + C$$

$$= \int \frac{t^2 - y}{t} dt = \int (t + y/t) dt$$

$$= \frac{t^2}{2} + y \ln|t| + C$$

~~When~~  $\frac{dy}{dt} = \frac{t^2}{2} + y \ln|t| + C$

Therefore.

$$\text{Then } y(t) = \int \left( \frac{t^2}{2} + y \ln|t| + c_1 \right) dt$$

$$= \frac{t^3}{6} + y(t \ln|t| - t) + c_2$$

$$= \frac{t^3}{6} + ty(\ln|t| - 1) + c_2$$